Power Spectral Analysis of Signal with Time- Varying Characteristics

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ملخص

لقد تم استخدام أساليب التحليل الطيفي المتغير في السنوات الماضية وبشكل متكرر على الاشارات غير المستديمة. في هذا البحث نقترح تجربة ومقارنة اثنان من خوارزميات شدة القوى للاشارة المباشرة لميزة الزمن - التردد للاشارة غير المستديمة. يستند الخوارزمي الأول في التحليل على ميزة الزمن - التردد المستمرة ويستند الثاني على ميزة الزمن - التردد المتقطعة تم تقييم أداء كل خوارزمي ومقارنته مع التحليل الطبيعي المتغير. بينت النتائج بأن التحليل المقترح يقدم أداء جيد جداً في كل من ميزات الزمن - التردد المستمرة والمتقطعة في حالة التقييم التقليدي وفي حفظ احصاءات المعلومات. تم حساب تحويلة فورير لشدة الاشارة في حالتي الشكل المستمر والمتقطع. كما تم اقتراح طريقتان لتطبيق الخوارزمي المقترح. اضافة لذلك تبين بأن هذا الخوارزمي مفيد لاكتشاف امكانية كون الاشارة غير مستديمة.

وقد تمت المقارنة مع التحليل المويجي وتبين أن لهذه الطريقة المقترحة نتائج أفضل في المجال الطيفي بينما أظهر التحليل المويجي ناتج أفضل في المجال الزمني.

ABSTRACT:

During the past two decades, the spectrogram method has widely used to analyze the characteristic of non-stationary signals. In this paper, two of the power intensity of the signal algorithms for time-frequency characteristic of non-stationary signal are proposed, tested and compared to standard schemes. The first algorithm is based on the continuous time-frequency characteristic and the second is based on the discrete time-frequency characteristic.

The performance of each algorithm is evaluated and compared to the existing standard spectrogram analysis. The results show that the proposed power intensity waveform analysis provides very good performance in both continuous and discrete time - frequency characteristic for classical evaluation criteria, and for the preservation of data statistics. The Fourier transform of the intensity of signals is computed in continuos and discrete form. Two methods for the digital implementation of the proposed algorithm are suggested and described.
Moreover, It is shown that this algorithm is useful for detecting possible nonstationarity of signals. Comparisons of the result with multiresolution analysis reveals that the prepared algorithm has better frequency resolution where as the wavelet transform shows higher time resolution.

I. Introduction

In the past couple of years, the newly developed theory of multiresolution analysis has opened new horizons for the application of the power spectral density by making it possible to generate highly sparse direct and accurate method. The aim of the research work of which this paper describes a part is to find a solution to the problem of classifying genuine algorithms, as algorithm represents the time-frequency characteristic more efficiently than others. A literature survey, conducted in order to shed some light on this particular subject, has yielded a sparsity of work addressing this matter, [1-2] indicating a need for further research. Simple power spectral density analysis has been used in various studies to determine the frequency components of the signals under consideration. Power intensity estimation techniques have proved essential to the version of advanced radar, sonar, communication, speech, biomedical, geophysical, and other data processing systems. The available power spectrum estimation techniques may be considered in a number of separate classes, such as, conventional or Fourier type methods. The object of Fourier analysis is to provide a connection between time functions, which are usually expressed as some type of waveform, and functions of frequency or spectra. The fast Fourier transform (FFT) is used in many modern spectrum analysis of waveforms, and this can result in a very high spectral resolution over a wide signal bandwidth, however, it remains of limited flexibility to analyze this type of problems due to the stationary assumptions. For signals with marked changes in time and frequency, it is important to study the time-frequency characteristics. To measure and quantify important features of time and frequency a sophisticated quantitative method is still needed. In general, time dependent spectrum, is simply a signal transformation that depends on two variables, namely time and frequency [3].

The spectrogram [1] is an old and obvious method for dealing with nonstationary signals. In the spectrogram, the signal under study is subdivided into smaller records and each subrecord is assumed to be stationary. Each subrecord is multiplied by a data window and then FFT- analysis is applied on each data record. Although an analysis of this type produces useful qualitative characteristics of the signal, careful attention should be paid in interpreting the quantitative feature of the plot. Depending on the duration of the data window averaged values of the fluctuations characteristics are obtained.

In this paper, we propose a new technique to study spectra of nonstationry signals. The technique depends on the F.T. of the intensity of the signal rather than the F.T. of the signal directly. This technique has the advantage over the spectrogram due to its simplicity and efficiency. A subdivision of the signal is not needed and hence, the limitation due to the time-frequency uncertainty is avoided. The proposed power density technique will be presented and its advantages over other existing spectrogram techniques will be discussed.

This paper focuses on the spectra of nonstationary signal. Section II out lines briefly the mathematical analysis that is used. In Section III the digital implementations of the proposed schemes are introduced and some suitable mathematical representations are given. In Section IV several results for the proposed algorithm are presented and compared with the spectrogram values that appear in the literature. The implementation of the wavelet transform is carried out by performing the convolution operation in the frequency domain as presented in Section VI. That is, for each given value of the scales parameter an the FFT is called three times. Conclusion and modern spectrum waveforms for nonstationary signal are presented in Section V.

II. Mathematical Analysis

This section provides an introduction to the tools and mathematical techniques necessary for understanding and analyzing both continuous-time-frequency characteristic and discrete-time-frequency characteristic of the nonstationary signal. In power spectrum estimation, the signal under consideration is processed in such a way that the distribution of power among its frequency components is estimated. As such, phase relations between frequency components are suppressed. The information contained in the power spectrum is essentially that which is present in the autocorreclation sequence, this would suffice for the complete statistical description of a Gaussian signal. The goal of this section is to introduce the time-frequency method to detect the magnitudes and possibly the phases of the signals. There are several methods which have historically been used. The Fourier transform method is probably the most widely used of the methods of determining magnitude and phase, and is highly accurate. For either transient or sinusoidal calculations, the complex magnitude of the wave can be calculated from the time-domain waveform using

$$G(k\Delta f) = \Delta t \sum_{n=0}^{N} g(n\Delta t) e \frac{-j2\pi kn\Delta t}{N} \qquad (1)$$

where

$G(k\Delta f)$ is the complex magnitude
$g(n\Delta t)$ is the time-domain waveform
Δf is the frequency resolution
Δt is the time resolution
n is the time step index = 0,1,2,..., N
K is the frequency index
N is the length of the Fourier transform = $1/(\Delta f\, \Delta t)$

For transient simulations, the simulation may converge, and all signal values, $g(n\Delta t)$ may go to zero before the summation in (1) is complete. In that case, the summation can be stopped before N summations, which saves computational time. For sinusoidal (single - frequency) applications, the summation is done for one cycle often the simulation has converged to steady - state.

The Fourier transform in (1) can be calculated with either the Fast Fourier Transform (FFT) or the Discrete Fourier Transform (DFT) in (1). It has been shown that the DFT is actually faster than the FFT for signal applications, although many people still use the FFT method because of the convenience of prepackage Fourier transform software. Both methods are equally accurate.

A. **Analysis in the Continuous Case:** In this section the Fourier transform for weakly non-stationary signal as a function of time and frequency is given by [4]

$$F\{s^2(t)\} = \int df\, S(f + \nu)\, S^*(f) \qquad (2)$$

We note that if the signal is stationary, the statistical averaging of the intensity waveform $s^2(t)$ is constant. In other words it follows from equation (2) that the statistical averaging of $S(f+\nu)S^*(f)$ must vanish for all values with $\nu \neq 0$. In a weakly nonstationary case we may thus write:

$$F_\nu\{E[s^2(t)]\} = \int df\, E[S(f + \nu)S^*(f)] \qquad (3)$$

where E denotes the expected value or statistical averaging. At this point we might, with good reason, ask "What have we gained?" We have derived for a weakly nonstationary case, equation (3) will not vanish. Hence, the Fourier transform will be a function of frequency and time at the same time indicating a nonstationary scharior of the signal. In the next section, we review important results, for a discrete-time random signal, because we are interested in digital signal processing applications and the data we work with its assumed to be sampled. The discrete-time Fourier transforms up the autocorrelation function are known as the power spectrum of the stationary signal and the cross-power spectrum between two real, stationary, zero-mean random signals.

This Fourier integral maps a signal in one domain (time) to another domain (frequency). Equation (3) provides a continuous output as a function of a time variable t for the continuous input signal as a function frequency variable f. Although the Fourier transform pair provides a powerful analytic tool for the analysis of continuous signals and systems, the implementation of the transform requires a mathematical representation of the function to be transformed and the calculation of the integral expression. It is desirable to perform a similar operation on discrete signals to facilitate implementation with digital hardware. (See Section II. B. for the definitions and analysis of the discrete case.)

B. Analysis in the Discrete Case: The development of discrete-time-frequency characteristic relationship is presented. Since, the frequency response of a signal is recognized as the Fourier transform of the system [4].

The intensity waveform in discrete form may be written as:

$$S^2(m) = \sum_{n}\sum_{n^1} S_n S^*_{n^1} e^{j2\pi m(n-n^1)/N} \quad (4)$$

and the discrete Fourier transform of $S^2(m)$ is defined as :

$$\mathrm{DFT}\{s^2(m)\} = \frac{1}{N}\sum_m S^2(m)e^{-j\pi mn/N}$$

$$= \sum_{\ell}\sum_{\ell^1} S_\ell S^*_{\ell^1} \quad \frac{1}{N}.\sum_m e^{j2\pi m(\ell-\ell^1-n)/N} \tag{5}$$

$$= \sum_{\ell} S_{\ell+n} S^*_{\ell^1}$$

This discrete-sum spectral representation for a finite-length sequence is referred to, appropriated, as the discrete Fourier transform (DFT).

If the fluctuation is nonstationary, then

$$E[S_{\ell+n}S^*_{\ell^1}] = 0, \qquad n \neq 0 \tag{6}$$

and hence we have:

$$\mathrm{DFT}\{E[S^2(m)]\} = \sum_{\ell} E[S_{\ell+n}S^*_{\ell^1}] \tag{7}$$

III. Digital Implementations

Fourier representations for signals proved useful in both providing spectral information about a signal as well as a frequency-domain tool for systems analysis [5]. Efficient algorithms for the computation of the DFT are desirable, and we consider the development of such algorithms in this section. The most sensitive issue in the digital implementation is the statistical averaging. We propose in [4] two methods for averaging, averaging over a duration time and over a set of decimated series.

A. Averaging Over a Duration Time $T_1 = m\,\Delta t$

The duration time procedure of (3) can be performed for the available proposed algorithm by the following steps:

1. Form $L = \frac{N}{M}$ sections of data $S^{\ell}(m)$, m = 1,...,M, and $\ell = 1,\dots,L$
2. Compute the mean square value averaged over M points.

$$E[S^2(\ell)] = \frac{1}{M}\sum_{m=(\ell-1)M+1}^{m=\ell M}[S(m)]^2 \tag{8}$$

3. Compute

$$P_\ell = SFT\{E[S^2(\ell)]\}, \qquad \ell = 1,\dots,L/2 \tag{9}$$

4. The mean square value spectrum of the intensity waveform is

$$I_\ell = 2|P_\ell|^2, \qquad \ell = 1,\dots,L/2 \tag{10}$$

B. Averaging Over a set of Decimated Series

Many methods for reducing the number of multiplications, usually more critical than the number of additions, have been investigated over the last 50 years. The most important technique, popularized and earlier reported in [6], is based on decomposing or breaking the total transform. This decimation process can be done in both the time and frequency domains. Time decimation can be used to significantly rescue the length of the sum in (1) and (7), and improve the computational efficiency of the algorithm. In the following presentation, the number of points is assumed as a power of 2, that is, N =2. The decimation - in - time approach is one of breaking the N-point transform into two (N/2) - point transforms, and continuing this process until two - point transforms are obtained. The first step of this process is described below. We now consider DFT sum (7) with the sum carried out over the first half and the last half of the input signal separately. This results in the sum by the following decimation procedures:

1. Form M series by decimating each M^{th} point ML = N

$S^{(1)}(\ell)$, $\ell = 1, M+1, 2M+1, \ldots, (L-1)\,M+1$
$S^{(2)}(\ell)$, $\ell = 2, M+2, \ldots, (L-1)M+2$

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$S^{(M)}\ell$, $\ell = M, 2M, \ldots, LM$

2. Compute the squared value

$[S^{(m)}]^2$, $m = 1, \ldots, M$ and $\ell = 1, \ldots, L$

3. Compute the DFT

$$\mathrm{DFT}\{[S^{(m)}(\ell)]^2\}, \quad m = 1, \ldots, M \text{ and } \ell = 1, \ldots, L/2$$

4. Compute the sample mean square value spectrum

$$I_\ell^{(m)} = 2|DFT\{[S^{(m)}(\ell)]^2\}|^2$$

5. Average over m sections

$$I_\ell = \frac{1}{M}\sum_{m=1}^{M} I_\ell^{(m)} \qquad \ell = 1,\ldots,L/2$$

Both methods are basically based upon the assumption that the statistical characteristics of s(t) is roughly stationary during an interval $M \Delta t$. In other words, both methods have the effective sampling interval $M \Delta t$ by averaging. That is, with respect to the frequency resolution both methods are identical. The two methods are however different and should be applied simultaneously depending on the application and the form of the fluctuation signal under study [6].

IV. Discussion

To evaluate the viability and the effectiveness of the proposed optimum power intensity, computer simulations was performed to compare various plots generated by the power intensity scheme and compared with the spectrogram. Figure 2 displays the contour plot. In Figures 3 and 4 we display the power intensity for the same time signal. Figure 3 displays the surface plot and Figure 4 the contour plot.

As it is demonstrated by the comparison of Figures (1,2) and Figures (3,4), the time-frequency resolution of the power intensity method is much more accurate than that of the spectrogram. A large DC value, due to squaring, is obtained in the proposed algorithm. Since this DC value has no physical significant, it can be removed easily using simple filtering technique. Also, it is only accurate when the simulation produces a clean, perfectly converged signal without DC offsets or noise.

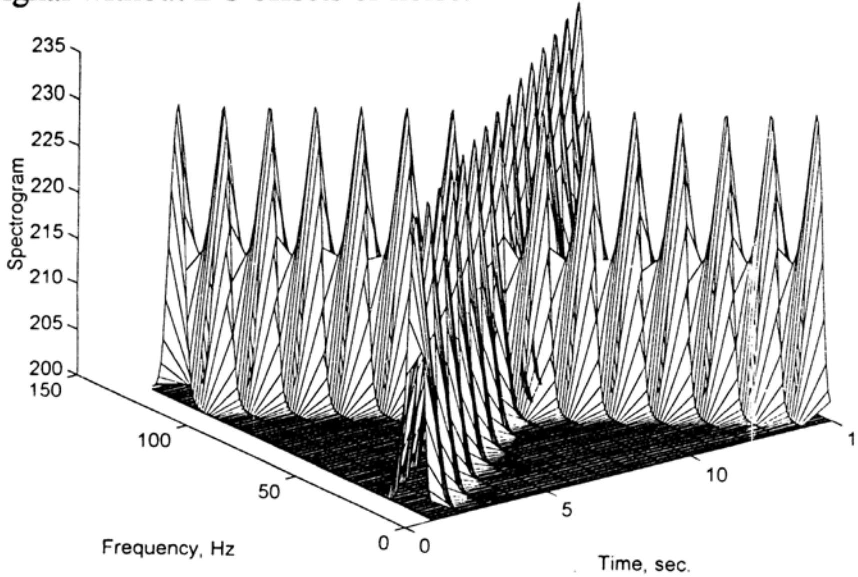

Figure 1 Spectrogram Surface Plot of Crossed-Chirp Signal.

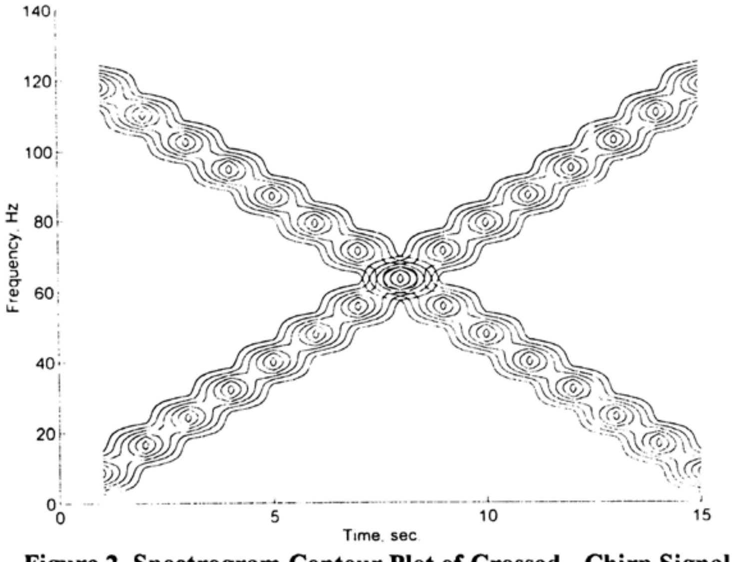

Figure 2 Spectrogram Contour Plot of Crossed - Chirp Signal.

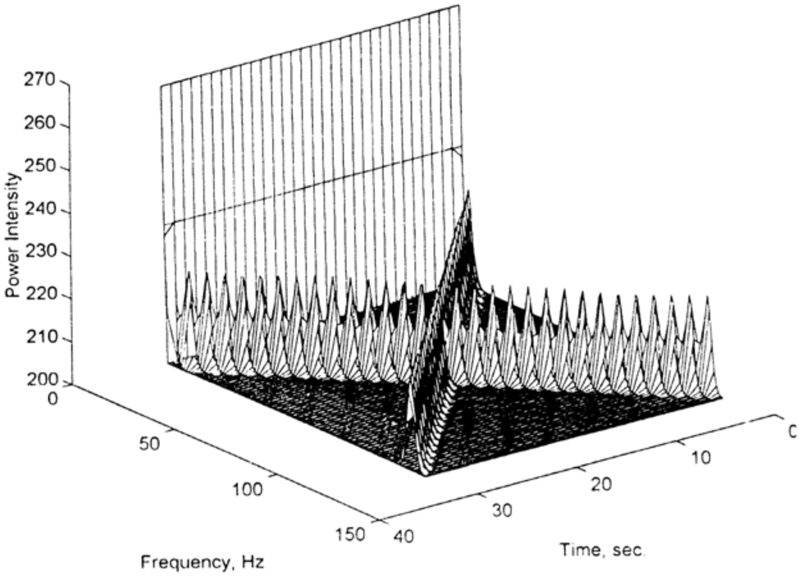

Figure 3 Power Intensity Surface Plot of Crossed - Chirp Signal.

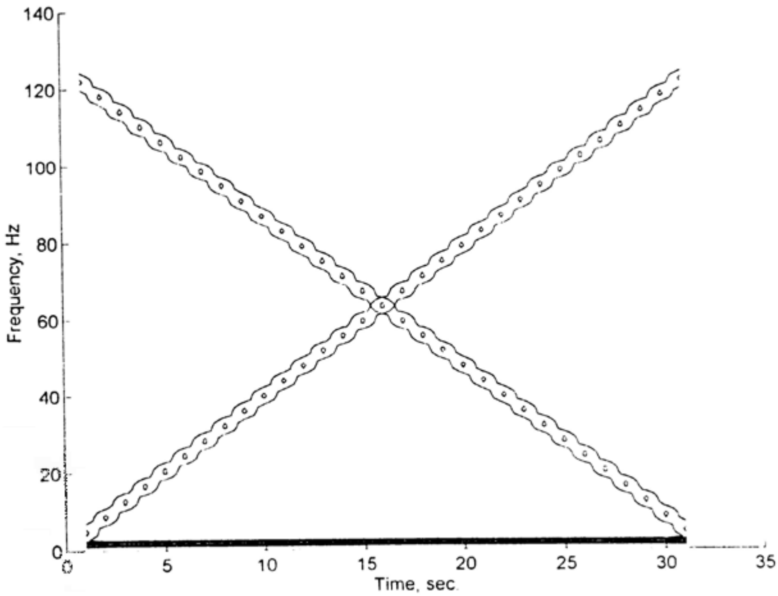

Figure 4 Power Intensity Contour Plot of Crossed - Chirp Signal.

VI. Implementing Wavelet Transforms

The Fourier, cosine and sine transforms are non-local. All the elements of their basis functions are active for all time t, and series expansions using such bases converge only by dint of enormous cancellations over almost an infinite number of terms. The time-frequency resolution properties of such transforms are limited by the Heisenberg uncertainty principle: $\Delta(t)\,\Delta(f) \geq 1/4\pi$, where $\Delta(t)$ and $\Delta(f)$ are the transform resolutions in the time and frequency domains respectively. The uncertainty principle suggests the possibility of trading $\Delta(t)$ and $\Delta(f)$. Wavelets present the most efficient means for doing so. The use localized basis functions and are thus capable of fielding good signal approximations with only a few terms. Because wavelets are localized within an interval, resolution in time and frequency domains can be traded, making it feasible to take a quick look at particular signal interval efficiency. The wavelet prototype function used for analysis is called the mother wavelet [7]. This function is dilated and translated to achieve the basis functions at different scales. Once the mother wavelet g(t) is fixed, a basis set consisting of its translations and dilations is formed. This choice gives a sparse bases and also facilitates multiresolution analysis.

A. Multiresolutions Analysis

Basically, the wavelet transform comprises a convolution of the input signal with an analyzing wavelet (window), scaled and shifted to provide a time-frequency distribution.

If s(t) is the signal to be analyzed and g(t) is the analyzing wavelet, then the Wavelet Transform (WT) is defined as:

$$W(t,a)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} s(t)g^*(\frac{t-\tau}{a})dt \qquad (11)$$

where g*(t) is the complex conjugate of g(t), a is the scale parameter, controlling the dilation of the window function, effectively stretching the window geometrically over time. The translation parameter t centers the window in the time domain. The geometric scale gives the wavelet transform a "zooming" capability over a logarithmic frequency range, such that high frequencies are localized by the window over short time scales, and low frequencies are localized over longer time scales.

The scaling factor a is defined as $a = f_0/f$ with the basic frequency $f_0 = \omega_0/2\pi$.

The resolution of the WT varies with the scale parameter a and, hence, with the frequency scanned. As the scale parameter decreases the resolution increases in the time-domain and decreases in the frequency domain. This variation in resolution gives the wavelet transform the capability to detect very small details and irregularities in the signal. To see accurate changes, transients, and abrupt changes in the signal, we can choose small values for the scale parameter a.

To demonstrate this fact we consider the following signal

$$x(t) = \sin(2\pi(10)t) + \delta \qquad (12)$$

with δ representing small distortion at time t. The distortion δ = 0.03 of the signal height. Figure 5 displays the signal s(t). As it is demonstrated in the figure, the small distortion is very difficult to detect with the naked eye.

Figure 6 shows the wavelet transform of the signal and Figure 7 shows the wave intensity. The wavelet transform can localize the location of the distortion very accurately, where as in the Figure 8 the localization is not possible.

There are many choices for mother wavelets, g(t), but should satisfy a certain number of properties. The most important are integrability, square integrability, and that it has no DC component. Moreover, it is convenient to assume that g(f) =0 for negative frequencies. In addition, the wavelet has to be concentrated in the time and frequency instantaneously. Hence a frequently used analyzing wavelent is

$$g(t) = e^{-t^2/z} \bullet e^{j2\pi w_0 t} \tag{13}$$

where w_0 is the frequency of the mother wavelet before it is scaled. This kernel does not meet "admissibility conditions" to reconstruct the signal from the $w_0(t,a)$ coefficients, in contrast to orthogonal basis function [7].

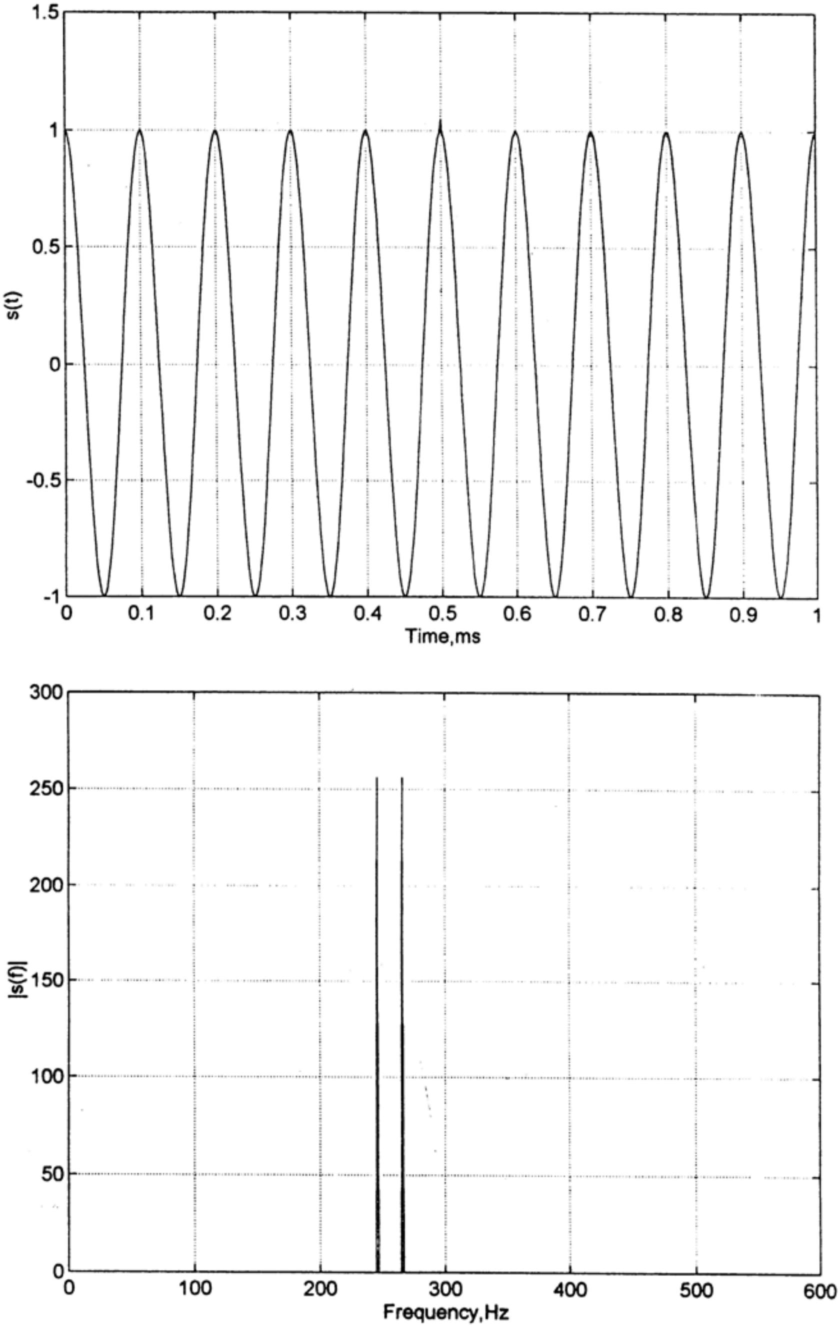

Figure 5 The Distorted Signal and its FFT

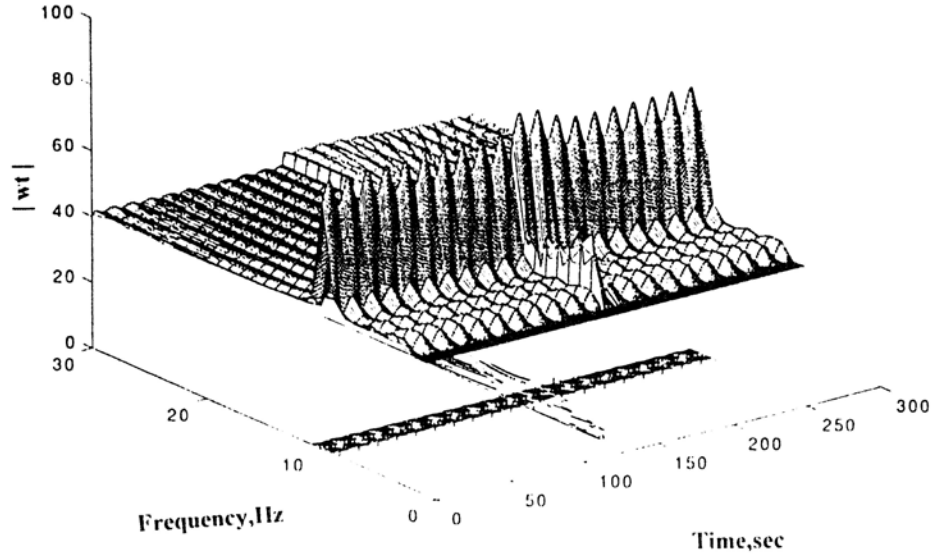

Figure 6 The Wavelet Transform of the Distorted Signal.

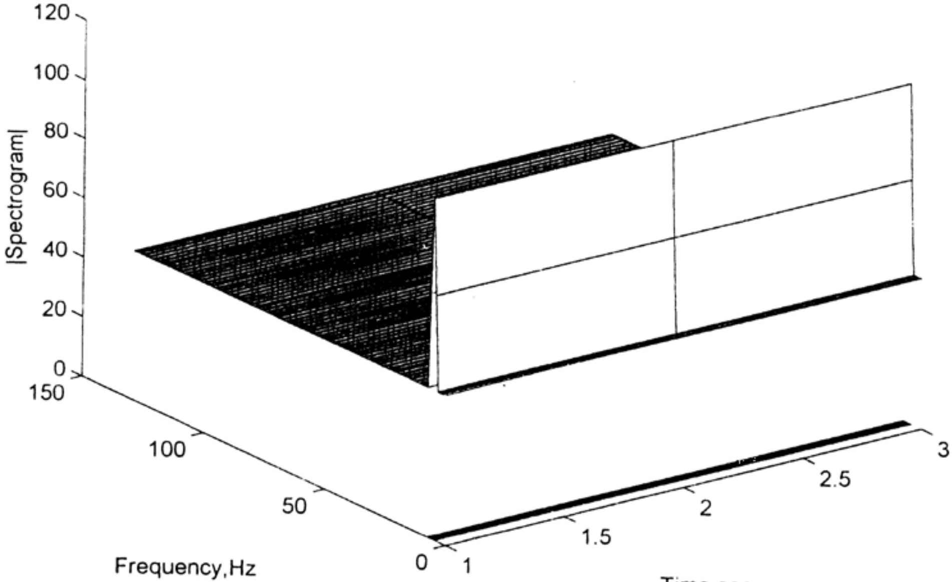

Figure 7 The Spectrogram of the Transient Gaussian Modulated Sine-Wave.

V. Conclusion and Further Work

A new method is proposed to describe the characteristics of nonstationary signals. The intensity or power of the signal is considered rather than the signal directly. We derive the theoretical algorithm and propose two procedures for the digital implementations. The two procedures have the same effective frequency resolution and hence are equivalent in that respect. They are, however, different in estimating the expected values and they should be used depending on the application form and the type of signal under study.

As this method of power intensity advised to spectral domain of the nonstationary signal data, it would seen likely that spatial or statistical spectrogram techniques could also be used on the nonstationary signal data. Previous study [1] has shown that, the subrecord is assumed to be stationary for the signal under study, this type of spectral data makes it very hard to analyze with standard spatial methods.

The most obvious choice of modern spectrum analysis of waveforms for this type of data of nonstationary signal seems to be that of wavelet analyzer. Hence, a power intensity and wavelet spatial spectrum method are proposed as the most likely path for attainary increased analysis.

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References

[1] M. R. Portnoff, Time-Frequency Representation of Digital Signals Based on Short Time Fourier Analysis, IEEE Trans. Acous. Speech sign. Proc. 28 (1980), pp.55-69.

[2] F. Hlwatsch and G. F. Boudreaux-Bartels, Linear and Quadratic Time Frequency Representation, IEEE Sig. Proc. Mag. April (1992), pp. 21-67.

[3] M. Alfaouri, Synthesizer to Speech Analysis Based on Pseudo Wigner Distribution, IEEE, ICECS, 1996, Rodos, Greece, October 13-16, 1996, Vol. 1, pp. (497-501)

[4] M. Alfaouri, Power Spectral Density of an Intensity Waveform, IEEE, ISDSP, 96, July 22-25, London, UK. pp. (118-121).

[5] M. Alfaouri, Digital Analysis of Signal Intensities, IEEE,TENCON, 1996, Perth, Westren Australia, Vol. 2, November 27-29, pp. (536-540).

[6] L. Ludeman, Fundamentals of Digital Signal Processing, Wiley, New York, 1986.

[7] O. Rioul and M. Vetterli, Wavelets and Signal Processing, pp. 14-38, October 1991.