Method of Conjugate Gradient and Fast Fourier Transform for Numerical Solution of Conducting Plate of Resonant Size

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ملخص

يقدم البحث وبالتفصيل توليفة جديدة تجمع بين طريقة تدرج المرافقة (CG) وتقنية تحويلة فوريير السريعة - الوقت اللازم لحل مشاكل (معضلات) التشتت المغناطيسي باستخدام هذه التوليفة أقل بكثير من الوقت اللازم باستخدام طريقة تدرج المرافق (CG) المألوفة باستخدام طريقة العزوم (MOM). وكذلك يتطلب تطبيق الحسابات الكهرومغناطيسية (ACE) حسابات دوال الجهد المعطاة بدلالة تكاملات التلفف «الطيات» التي يمكن حسابها بشكل فعال باستخدام التوليفة (CG-FET). تمت طريقة العمل بسهولة وبالترتيب وذلك باستخدام عملية العينات ودوال Roof-Top لتمثيل التيار المتولد والنبضات وذلك للحصول على معدل الفيض. يعتبر المخطط وسيلة تحليلية فعالة لأن به تم استبدال المشتقات في حقل التحويل بعمليات ضرب بسيطة مع غياب (أو عدم ظهور) بعض المعضلات الحسابية التي تظهر باستخدام الطريقة المألوفة (CG) أو باستخدام طريقة العزوم (MOM). يتجنب هذا المخطط بالمقارنة مع طريقة العزوم خزن المصفوفات الكبيرة ويقلل زمن الحسابات بالحاسوب وذلك بترتيب القيم. لذلك تحتل هذه الطريقة كهربائياً تركيبات صغيرة ويمكن استعمالها بسهولة. وأخيراً وبما أن الطريقة تكرارية فإنه من الممكن معرفة دقة حل المعضلة (أو المشكلة). لقد تم التحليل باستخدام مستوى مربع مرات قابلية توصيل عالية. لقد تم عرض النتائج ومقارنتها مع القيم

التحليلية، الرقمية أو المقاسة والمنشورة في مقالات سابقة «بحوث سابقة». كما أن تفاصيل المعادلات الواصفة للمخطط وطريقة الحسابات قد تم عرضها بشكل كامل مع عرض للنتائج الرقمية للمعضلة.

ABSTRACT:

Simple and efficient novel combination of the conjugate gradient (CG) method with the fast Fourier transform technique (FFT) is presented. With this combination, the computational time required to solve electromagnetic scatterer problems is much less than the time required by the ordinary (CG) method and the method of moments (MOM). Also, applied computational electromagnetics (ACE) require the computation of potential functions given in terms of convolution integrals which can be calculated very efficiently by using the (CG-FFT). The procedure is made easy and systematic by using a sampling process with roof-top functions to represent the induced current and pulses to average the fields. The scheme is an efficient numerical tool since the spatial derivatives are replaced with simple multiplications in the transformed domain, some of the computational difficulties present in the ordinary (CG) method and the (MOM) do not exist here. Therefore, electrically small structures can also be handled more easily. In comparison with (MOM), this scheme avoids the storage of large matrices and reduces the coputer time by orders of magnitude. Finally, since the method is iterative, it is possible to know the accuracy in a problem solution. A perfectly conducting square plate is analyzed. Results are presented and compared with analytical, mumerical, or measured values that appear in the literature. The details of the formulation and the computational procedure are presented along with the numerical reslts for the problem.

I. INTRODUCTION

Electromagnetic scattering problems involving bodies of resonant size, with dimensions of only a few wave-lengths, and arbitrary geometry have a practical interest but simultaneously they challenge today's numerical methods. The method of mements (MOM), perhaps the most popular numerical tool due to its flexibility to analyze complex geometries, becomes computationally too expensive to analyze this class of problems (too much memory and CPU time are required with ordinary computers). On the other hand, high-frequency techniques are generally not accurate in here, and they are directly suitable only for a few canonical geometrys. These problems are resolved here by a combination of the conjugate gradient (CG) method and the fast Fourier transform (FFT). The main contribution of this paper is to demonstrate the superior performance of this novel combinations. Recent investigations to solve these type of problems have turned to the iterative methods, the most promising being the conjugate gradient (CG) method [1]-[3], that is used for problems formulated in terms of operator equations such as

$$LI = Y \tag{1}$$

where L is an integerodifferential operator, and I and Y are the unknown and excitations functions, respectively.

The central idea of the CG-FFT method in applied computational electromagnetics (ACE) is to solve the operator equation of an linear and time-invariant (LTI) problem imposing the boundary conditions in the spatial domain and computing the convolutions by means of the FFT [2]. An initial estimate of the solutions is made in the CG-FFT method, but neither the accuracy of the method nor the speed of convergence depends on this initial guess. At each iteration of the CG it is required to compute LP or $L^A P$, where P is a known function and L^A is the adjoint operator of L. Usually, these operations are convolution integrals that can be performed efficiently using the fast Fourier transform (FFT) [3]-[7]. These convolutions involve the 3D free space scalar Green's functions as shown further in this paper. To the author's knowledge only [6]-[8] indicate in some detail the procedure to analyze these problems using CG-FFT. In [6] convolutions are carried out using the analytical expression of the Fourier transform (FT) of the Green's function that extends over all space. Therefore, this FT has an inverse that is not limited in band, and if it is sampled at finite intervals as in [6], some "aliasing" problems can arise [9]. In [7] the Grean's function is discretized and made periodic prior to be transformed using FFT, thereby avoiding this "aliasing". The author's experience indicates that, using his approach, the best was to avoid aliasing and other errors, the period of the equivalent period problem (EPP) should be at least twice the length of the interval over which the potential function must be computed. The procedure described in this reference is simple and works well in situations that are not numerically complex, such as the static examples that are given, but fails to produce reliable results for electrodynamics problems formulated in terms of a more complicated integral equation such as the electric-field integral equation (EFIE), with a nonself-adjoint operator and second derivatives.

In this paper the procedure presented in [7] is refined introducing suitable functions to represent numerically the current, using a more sophisticated and systematic field-averaging process but maintaining the cyclic-convolution scheme of [7]. In particular, roof-top functions and pulses [10] have been chosen as basis and testing functions, respectively, in the process of discretizing the operator. It has been shown in [11] that these functions can lead to very systematic and efficient procedures for iterative methods. The author's experience indicates that, using his approach, the best choice for the basis and testing functions for this problem are bidimensional rooftop and razor blade functions, respectively.

This paper focuses on the electrodynamic problem of square conducting plate of resonant size. Section II outlines briefly the particular CG algorithm that is used. In Section III, a discrete operator is obtained from EFIE by averaging the fields using pulse functions. In Section IV, the basis functions to represent the current and fields are introduced. Section V details how to apply this operator efficiently doing cyclic convolutions with FFT. In Section VI, it is first indicated how to compute the inner product that appears in CG algorithm and then the adjoint operator. Finally, in Section VII , several results for the induced current and convergence rate are presented and compared with the theoretical or measured values that appear in the literature.

II. CG ALGORITHM

To solve the operator equation (1), the CG algorithm f arbitrary operators that appears in [2] can be considered. Th algorithm minimizes the functional

$$F(J) = \langle r,r \rangle = \|r\|^2 \qquad (2)$$

with r defined as

$$r = E - LJ_1 \qquad (3)$$

where J_1 is the approximate solution for (1) and <r,r> denotes the inner product. Following [2], the algorithm starts with an initial guess J_0 (typically, $J_0 = 0$) and generates.

$$P_0 = L^A r_0 = L^A (E - L J_0) \tag{4}$$

an iterative cycle is as follows

$$J_n = J_{n-1} + a_n P_n \tag{5}$$

$$r_n = r_{n-1} - a_n L P_n \tag{6}$$

$$P_n = L^A r_{n-1} + \gamma_{n,n-1} P_{n-1} \tag{7}$$

$$a_n = \frac{\| L^A r_{n-1} \|^2}{\| LP_n \|^2} \tag{8}$$

$$\gamma_{n,n-1} = \frac{\| L^A r_{n-1} \|}{\| L^A r_{n-2} \|} \tag{9}$$

To evaluate the relative error at the Kth iteration, the following expression can be used :

$$e_r = \frac{\| E - LJ \|}{\| E \|} \tag{10}$$

Observing the operations in each cycle of (5)-(9), it can be noted that the more CPU - time - consuming computations are those of the type Lw, $L^A w$ that can be seen as the electric field created by source w using the operator L or its adjoint L^A. There are several advantages of this approach. The computations involve convolution integrals in the problem considered here are performed efficiently by using the FFT. The CG-FFT scheme proposed here enjoys another advantage of using Fourier transform, such as the simplification of the complex operations like derivatives. Also for the scheme proposed here, the CG version chosen guarantees monotonic convergence. This means that the relative mean square error always decreases with the number of iterations.

III. BASIS FUNCTIONS AND FORMULATIONS OF THE PROBLEM.

In Figure 1 a problem with square conducting plate is indicated. The plate can be inscribed in a square of sides L_x, L_y. This square is divided by a grid of $N_x \times N_y$ straight lines parallel to the coordinate axes into $(N_x+1) \times (N_y+1)$ square patches of area Δx Δy, where $\Delta x = L_x/(N_x+1)$ and $\Delta y = L_y/(N_y+1)$. A perfectly conducting square plate is illuminated in the direction of the local angles (θ^i,ϕ^i) by an incident field E^i created by a source current J^i anywhere in space. This field will induce currents on the plate J, which will generate a scattered field E^S. At any point in space, the total field E^T will be the sum of the incident and scattered fields, assuming that no other sources exist. The scattered field will depend on the particular currents induced on the plate. The scattered field will depend on the particular currents induced on the square conducting plate. Thus, the first step consists of obtaining these induced currents.

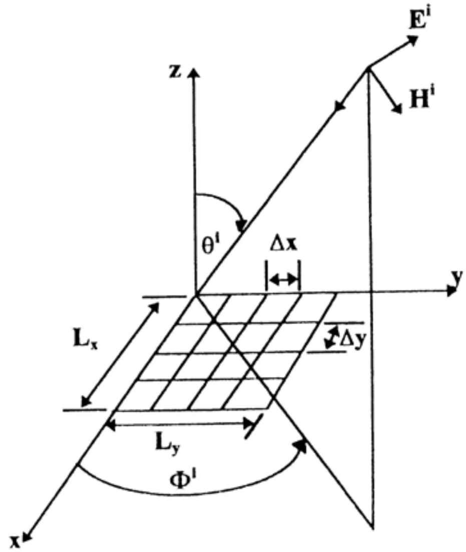


Figure 1 Schematic representation of the square-conducting-plate.

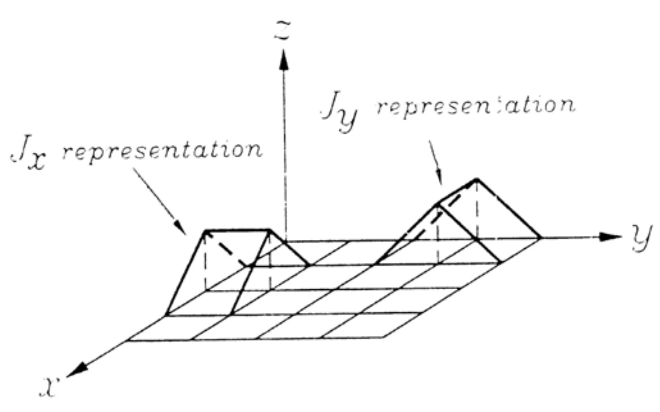


Figure 2 Basis functions used to represent the components of the current.

The electric fields or currents in the plates are approximated in terms of roof-top functions [10] and [11]. The shape and position of these roof-tops with respect to the discretization cell is shown in Figure 2, and the mathematical form of the currents will be

$$J_x(x,y) = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y+1} J_x^D[m,n] T_{mn}^x(x,y) \tag{11}$$

$$J_y(x,y) = \sum_{m=1}^{N_x+1} \sum_{n=1}^{N_y} J_y^D[m,n] T_{mn}^y(x,y) \tag{12}$$

with the roof-tops given by

$$T_{mn}^x(x,y) = \begin{cases} 1 - \dfrac{|x - \Delta x|}{\Delta x} & \text{for } (m-1)\,\Delta x < x < (m+1)\Delta x \\ & \text{and } (n-1)\Delta y < y < n\Delta y \\ 0 & \text{elsewhere} \end{cases} \tag{13a}$$

$$T_{mn}^y(x,y) = \begin{cases} 1 - \dfrac{|y - \Delta y|}{\Delta y} & \text{for } (n-1)\Delta y < y < (n+1)\Delta y \\ & \text{and } (m-1)\Delta x < x < m\Delta x \\ 0 & \text{elsewhere} \end{cases} \tag{13b}$$

The centers of the roof-tops are located at:

$$r_x^{mn} = m\Delta x\boldsymbol{x} + (n - 0.5)\,\Delta y\boldsymbol{y} \tag{14a}$$

$$r_y^{mn} = (m - 0.5)\,\Delta x\boldsymbol{x} + n\Delta y\boldsymbol{y} \tag{14b}$$

The discrete functions $J_x^D(m,n)$ and $J_y^D(m,n)$ in (11) and (12) must be zero for the roof-top located outside the square plate and to smooth the fields around the point in the direction in which a second derivative occurs. To take this into account, we define the discrete functions $W_{mn}^x(x,y)$ and $W_{mn}^y(x,y)$ to be equal to zero if the corresponding roof-top function is located outside the square plate and equal to unity otherwise. Such functions are usually called razor blade functions and are expressed for the x-and y-components as

$$W_{mn}^x(x,y) = \begin{cases} 1 & \text{if } \begin{cases} (m-0.5)\Delta x < x < (m+0.5)\Delta x \\ y = 0.5n\Delta y \end{cases} \\ 0 & \text{elsewhere} \end{cases} \tag{15a}$$

$$W_{mn}^y(x,y) = \begin{cases} 1 & \text{if } \begin{cases} x = 0.5m\Delta x \\ (n-0.5)\Delta y < y < (n+0.5)\Delta y \end{cases} \\ 0 & \text{elsewhere} \end{cases} \tag{15b}$$

The centers of these functions are the same as those in (14), respectively.

The testing procedure consists of computing the integral of the field component (incident and scattered) multiplied by the corresponding weighting functions. As will be seen later, only the first-order derivatives have to be computed, because this integral is canceled out by the partial derivative that is in the same direction as the weighting function

$$V_x(x,y) = \int_s W_{mn}^{x}(x,y)E_x(x,y)ds \tag{16a}$$

$$V_y(x,y) = \int_s W_{mn}^{y}(x,y)E_y(x,y)ds \tag{16b}$$

IV. DISCRETIZATION OF THE INTEGRAL EQUATION

The scattered field will depend on the particular currents induced on the plate. Thus, the first step consists of obtaining these induced currents. This is done, by enforcing the total tangential electric field on the plate to zero and noticing that the tangential magnetic field component in the system is related to the surface current density on both sides of the plate (surface formulation). The 3D free space scalar Green's function can be formulated in terms of the scalar wave equation. For this problem, the mixed potential integral equation (MPIE), explicitly considering the potentials due to both currents and charges to be solved at the surface patch z = 0, is given as

$$E_x^i(r) = k^c \int J_x(r')G(r,r')ds' + k^q \frac{\partial}{\partial x}\int\left(\frac{\partial J_x}{\partial x'} + \frac{\partial J_y}{\partial y'}\right)G(r,r')ds' \tag{17a}$$

$$E_y^i(r) = k^c \int J_y(r')G(r,r')ds' + k^q \frac{\partial}{\partial y}\int\left(\frac{\partial J_y}{\partial y'} + \frac{\partial J_x}{\partial x'}\right)G(r,r')ds' \tag{17b}$$

The values of the constants are

$$k^{c} = j\omega\mu_0 \tag{18}$$

$$k^{q} = -\frac{1}{j\omega\varepsilon_0} \tag{19}$$

The solution to the scalar Green's function is the well-known expression

$$G(r,r') = \frac{\exp[-jk_0|r-r'|]}{4\pi|r-r'|} \tag{20}$$

where $k_0 = 2\pi/\lambda_0$ represents the wave number of the vacuum, r represents the observation point vector, and r′ the source point vector, in which a unit delta function is located producing a field G(r,r′) with r and r′ dependence.

The integrodifferential equation (17) can be rewritten in compact matrix form as

$$\begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = k^c \begin{bmatrix} L_{xx}^c J_x \\ L_{yy}^c J_y \end{bmatrix} + k^q \begin{bmatrix} L_{xx}^q & L_{xy}^q \\ L_{yx}^q & L_{yy}^q \end{bmatrix} \begin{bmatrix} J_x \\ J_y \end{bmatrix} \tag{21}$$

These can also be expressed as

$$E_x^i = -E_{xx}^s - E_{xy}^s \tag{22a}$$

$$E_y^i = -E_{yx}^s - E_{yy}^s \tag{22b}$$

where E_x^i and E_y^i are the tangential components of the impressed field on the surface of the plate S and, for instance, E_{xy}^s is the x-component of the electric field computed from J_y.

Equations (17) and (21) constitute the integrodifferential operator equation to be solved in order to obtain the induced currents on the structure. All fields in (22) are approximated by the roof-top functions defined in Section III. Equation (22) can be discretized as

$$E_x^{iD}(x,y) = -E_{xx}^{sD}(x,y) - E_{xy}^{sD}(x,y), \qquad \text{for } W_{mn}^x(x,y) = 1 \quad (23a)$$

$$E_y^{iD}(x,y) = -E_{yx}^{sD}(x,y) - E_{yy}^{sD}(x,y), \qquad \text{for } W_{mn}^y(x,y) = 1 \quad (23b)$$

It is assumed that the impressed field is known and thus the discrete functions E_x^{iD} and E_y^{iD} can be simply found using the continuos functions

$$E_x^{iD}(x,y) = \int_{(m-0.5)\Delta x}^{(m+0.5)\Delta x} E_x^i(x, y_x^D)dx \quad (24a)$$

$$E_y^{iD}(x,y) = \int_{(n-0.5)\Delta y}^{(n+0.5)\Delta y} E_y^i(x_y^D, y)dy \quad (24b)$$

with y_x^D and x_y^D are defined as

$$y_x^D = (n-0.5)\Delta y \quad (25a)$$

$$x_y^D = (m-0.5)\Delta x \quad (25b)$$

With these simple considerations, we can write the final expression of the operator equations as

$$\begin{bmatrix} E_x^{iD} \\ E_y^{iD} \end{bmatrix} = k^c \begin{bmatrix} L_{xx}^{cD} J_x^D \\ L_{yy}^{cD} J_y^D \end{bmatrix} + k^q \begin{bmatrix} L_{xx}^{qD} & L_{xy}^{qD} \\ L_{yx}^{qD} & L_{yy}^{qD} \end{bmatrix} \begin{bmatrix} J_x^D \\ J_y^D \end{bmatrix} \tag{26}$$

V. COMPUTATION OF THE INDUCED FIELDS USING CYCLIC CONVOLUTIONS AND THE FFT

This section deals with the discretization of the operator equation and subsequent discretization of the Green's function. The best choice for the basis and testing functions for this problem are bidimensional roof-top and razor blade functions, respectively. Bearing this in mind, we carry out the discretization of the operator equations for the inductive and capacitive terms due to the x-components and y-components of the currents. Consider first the inductive term due to the x-component of the current

$$\begin{aligned} E_{xx}^c[x,y] &= L_{xx}^c J_x \\ &= \int_s \sum_{m'=1}^{N_x} \sum_{n'=1}^{N_y+1} J_x^D[m',n'] T_{m'n'}^x(x',y')\, G(x,y;x',y') dx' dy' \end{aligned} \tag{27}$$

where $J_x^D[m',n']$ represents the set of unknown coefficients of the amplitude of each roof-top function. Note that J_x^D constitutes a set of discrete values only dependent on m' and n'.

To compute (27), we must evaluate the integral. A very useful technique for computing this integral is to approximate the roof-top with a pulse function with the same volume [11,12]. Introducing this, we obtain

$$E_{xx}^{Dc}[m,n] = \sum_{m'=1}^{N_x} \sum_{n'=1}^{N_y+1} J_x^D[m'n'] \int_{D_{m'n'}} G(x_x^D, y_x^D; x', y')dx'dy' \tag{28}$$

where $D_{m'n'}$ is the integration domain defined as

$$D_{m'n'} = \begin{cases} (m'-0.5)\Delta x < x' < (m'+0.5)\Delta x \\ (n'-1)\Delta y < y' < n'\Delta y \end{cases} \tag{29}$$

To obtain the last expression in (28) we have assumed that the integrand is constant over the interval Δx and equal to its midpoint value. By using the discrete Green's function, we can write the operator of (28) as discrete linear convolutions. Equation (28) becomes

$$E_{xx}^{Dc}[m,n] = \sum_{m'=1}^{N_x} \sum_{n'=1}^{N_y+1} J_x^D[m',n'] \; G^D[m-m', n-n'] \tag{30}$$

This discrete convolution can be expressed in a more compact form as

$$E_{xx}^{Dc}[m,n] = J_x^D[m',n'] * G^D[m,n] \tag{31}$$

The discrete linear convolution (31) can be evaluated efficiently be means of a cyclic convolution using FFT. As indicated in [6], we have

$$E_{xx}^{Pc}[m,n] = J_x^P[m,n] \otimes G^P[m,n] \tag{32}$$

and the symbol $\otimes$ indicates a cyclic convolution.

Where J_x^P and G^P are the periodic functions obtained by completing a period of length $(2N_X + 2,\ 2N_Y + 2)$ by padding with zeros. The new source function is defined as

$$J^P[m,n] = \begin{cases} J^D[m,n] & \text{for } 0 < m \le N_x \\ & \text{and } 0 < n \le N_y + 1 \\ 0 & \text{elsewhere} \end{cases} \tag{33}$$

and the new periodic Green's function G^P defined as

$$G^P[m,n] = G^D[m,n] \ \text{ for } \ |m| \le N_x + 1 \ \text{ and } \ |n| \le N_y + 1 \tag{34}$$

Utilizing (16), with this formulation, the inductive term of (32) can be computed efficiently using the following discrete periodic operator

$$V_{xx}^D[m,n] = \frac{k^c \Delta x}{N_x^T N_y^T} FFT^{-1} \{\tilde{\tilde{J}}_x^P[p,q] \tilde{\tilde{G}}^P[p,q]\} + \frac{k^q}{\Delta x N_x^T N_y^T} FFT^{-1}\{(F_x[p]-1)(1-F_x^*[p]\tilde{\tilde{J}}_x^P[p,q]\tilde{\tilde{G}}^P[p,q]\} \tag{35a}$$

Operating in the same way to compute the capacitive term due to the x-component of the current V_{xy}^{D} , the capacitive term due to the y-component of the operator equation V_{yx}^{D}, and the inductive term due to the y-component of the operator equation V_{yy}^{D}, respectively. We have

$$V_{xy}^{D}[m,n] = \frac{k^{c}}{\Delta y N_x^T N_y^T} \bullet \mathrm{FFT}^{-1}\{(F_x[p]-1)(1-F_y^{*}[q])\,\approx{J}_y^{P}[p,q]\,\approx{G}^{P}[p,q]\} \tag{35b}$$

$$V_{yx}^{D}[m,n] = \frac{k^{c}}{\Delta x N_x^T N_y^T} \bullet \mathrm{FFT}^{-1}\{(F_y[q]-1)(1-F_x^{*}[p])\,\approx{J}_x^{P}[p,q]\,\approx{G}^{P}[p,q]\} \tag{35c}$$

$$V_{yy}^{D}[m,n] = \frac{k^{c}\Delta y}{N_x^T N_y^T}\mathrm{FFT}^{-1}\{\approx{J}_y^{P}[p,q]\,\approx{G}^{P}[p,q]\} + \frac{k^{c}}{\Delta y N_x^T N_y^T}\mathrm{FFT}^{-1}\{(F_y[q]-1)(1-F_y^{*}[q]\,\approx{J}_y^{P}[p,q]\,\approx{G}^{p}[p,q] \tag{35d}$$

where

$$N_x^T = 2(N_x+1) \tag{36}$$

$$N_y^T = 2(N_y + 1) \tag{37}$$

In (35), the shift property of the discrete Fourier transform (DFT) has been employed to compute the finite differences of the capacitive terms [4]. The exponentials due to this property have been denoted by

$$F_x[p] = \exp\left(j\frac{2\pi}{N_x^T}p\right) \tag{38}$$

and

$$F_y[q] = \exp\left(j\frac{2\pi}{N_y^T}q\right) \tag{39}$$

$F_x^*[P]$ is the complex conjugate of $F_x[P]$, and $\underset{\approx}{J}_x^P$, $\underset{\approx}{G}^P$ indicate the periodic discrete Fourier transform of J_x^P and G^P, respectively.

VI. COMPUTATIONS WITH THE ADJOINT OPERATOR

The application of a CG scheme requires the use of the adjoint operator defined in terms of the inner product [13]

$$\langle A, LJ \rangle = \langle L^A A, J \rangle \tag{40}$$

where L^A denotes the adjoint operator of L. The inner product is now defined over the 2D vector functions A and J, with L representing the matrix operator equation in (21) and by imposing condition (40) on two arbitrary vectors A and J in the range of the operator (definition domain), we obtain the well-known results,

$$L^{A} = (L^{T})^{*} = \text{Hermitian of L} \tag{41}$$

in terms of the complex conjugate (*) of the transpose (superscript T).

The vector inner product of (40) is satisfied when we have

$$<A_x, L_{xx} J_x> = <L_{xx}^{A} A_x, J_x> \tag{42a}$$

$$<A_x, L_{xy} J_y> = <L_{yx}^{A} A_x, J_y> \tag{42b}$$

$$<A_y, L_{yx} J_x> = <L_{xy}^{A} A_y, J_x> \tag{42c}$$

$$<A_y, L_{yy} J_y> = <L_{yy}^{A} A_y, J_y> \tag{42d}$$

Using $E^{sA} = L^{A} J$ together with (40) and (42), we have

$$\begin{aligned} V_{xx}^{DA}[m,n] = {} & \frac{k^{c^*}\Delta x}{N_x^T N_y^T} FFT^{-1}\{\overset{\approx}{J}{}^{P}_{x}[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} \\ & + \frac{k^{q*}}{\Delta y N_x^T N_y^T} FFT^{-1}\{(F_x^{*}[p]-1)(1-F_x[p])\,\overset{\approx}{J}{}^{P}_{x}[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} \end{aligned} \tag{43a}$$

$$V_{xy}^{DA}[m,n] = \frac{k^{q*}}{\Delta x N_x^T N_y^T} \bullet \quad \text{FFT}^{-1}\{(F_y^*[q]-1)(1-F_x[p])\,\overset{\approx}{J}{}_y^P[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} \tag{43b}$$

$$V_{yx}^{DA}[m,n] = \frac{k^{q*}}{\Delta y N_x^T N_y^T} \bullet \quad \text{FFT}^{-1}\{(F_x^*[p]-1)(1-F_y[q])\,\overset{\approx}{J}{}_x^P[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} \tag{43c}$$

$$V_{yy}^{DA}[m,n] = \frac{k^{c^*}\Delta y}{N_x^T N_y^T}\text{FFT}^{-1}\{\overset{\approx}{J}{}_y^P[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} + \frac{k^{q*}}{\Delta y N_x^T N_y^T}\text{FFT}^{-1}\{(F_y^*[q]-1)(1-F_y[q])\,\overset{\approx}{J}{}_y^P[p,q]\,\overset{\approx}{G}{}^{P*}[p,q]\} \tag{43d}$$

VII. RESULTS

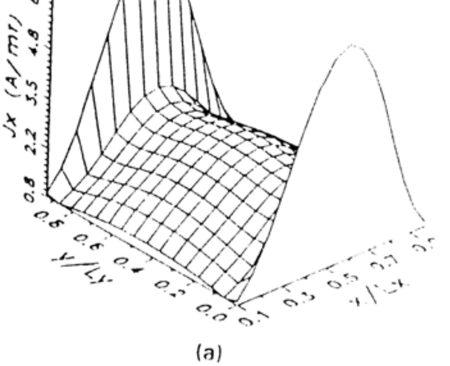
By using (35) and (43) for the direct and adjount operators, respectively, in the CG algorithm, we are able to obtain the following results:

- 2D representation of the magnitudes of the current;
- Convergence study for the components of the current;
- Convergence curves of the mean squared error,

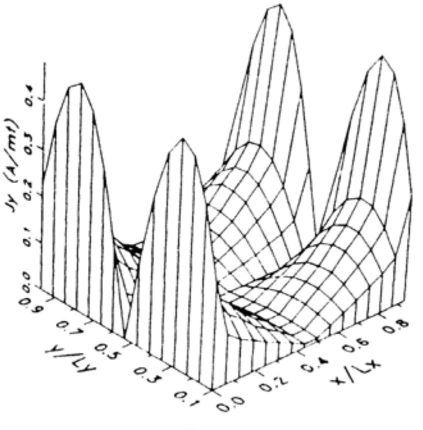
- Current distribution for an incident plane wave.

A. 2D Current Magnitudes

Figure 3 shows the two current components obtained from the CG-FFT scheme previously described for an x-polarized incident field ($\theta^i = 0$ deg and $\phi^i = -90$ deg). The number of samples used to represent the current was $N_x = N_y = 15$.



(a)



(b)

Figure (3a) 2D representation of the magnitude of the x-component of the current: $L_x = L_y = L = 1\lambda$.

Figure (3b) 2D representation of the magnitude of the y-component of the current: $L_x = L_y = L = 1\lambda$.

B. Convergence Curves

Figures 4 and 5 present a convergence study for the same structure in terms of the number of samples. The first two figures show the magnitude of the x-components of the current for two different cuts (AA′ and BB′) in the x- and y- directions, respectively. Figure 6 plots the relative error versus the number of iterations. The curves are representative of typical convergence rates obtained by using a CG scheme combined with the FFT in the way previously described.

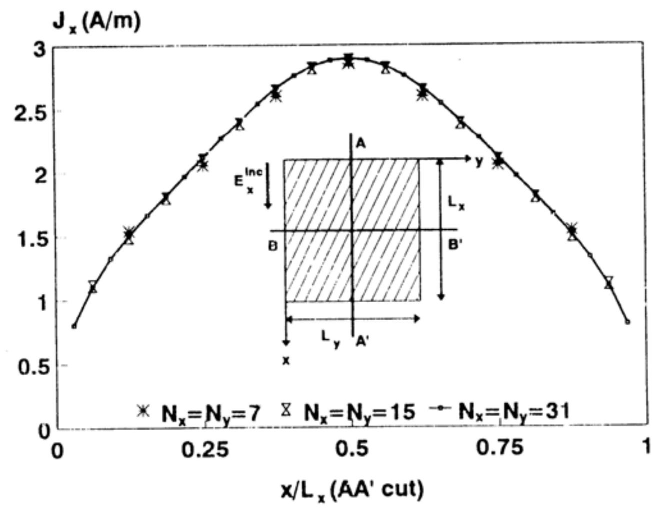


Figure 4 Convergence study for the x-component of the current along the AA′ cut.

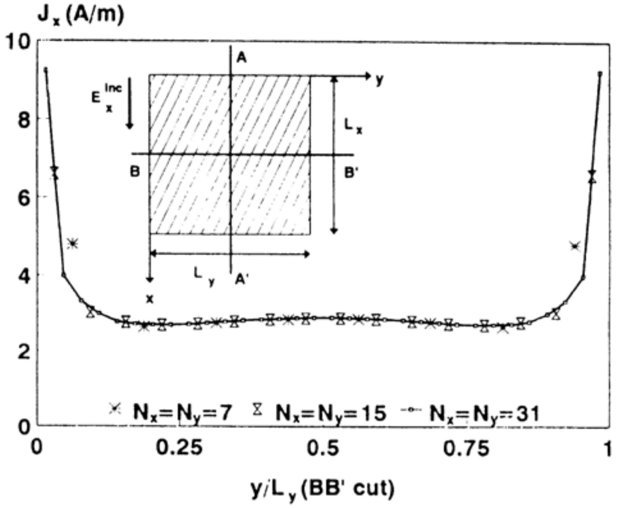


Figure 5 Convergence study for the x-component of the current along the BB′ cut.

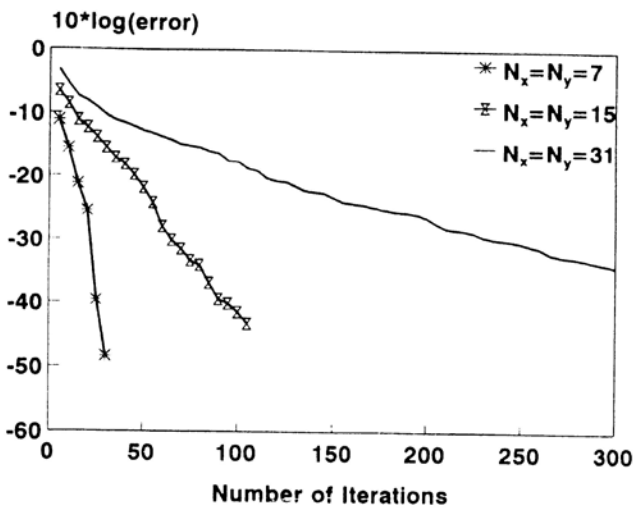


Figure 6 Relative error versus number of iterations in terms of the number of samples.

C. Current Distributions

Figure 7 shows the x-component of the current compared with the results obtained by Glisson and Wilton [12] using an MOM procedure. Cuts AA′ and BB′ are indicated in Figures 4 and 5. In this case, the number of samples was $N_x = N_y = 7$. The CPU time for this case was about 8 to 10 seconds using PC-486 (33 MHz, 4 MB), which gives a good idea of the power of the scheme under consideration. The computation time per iteration T_i follows the equation $T_i = AN_s \log N_s$, N_s being the number of samples considered to represent the unknown function, and A is a constant. The total computation time to the problem presented here T follows the law $T = AN_iN_s \log N_s$, N_i being the number of iterations considered. It can be found that the number of iterations required to solve (1) can be as great as the number of samples N_s [13].

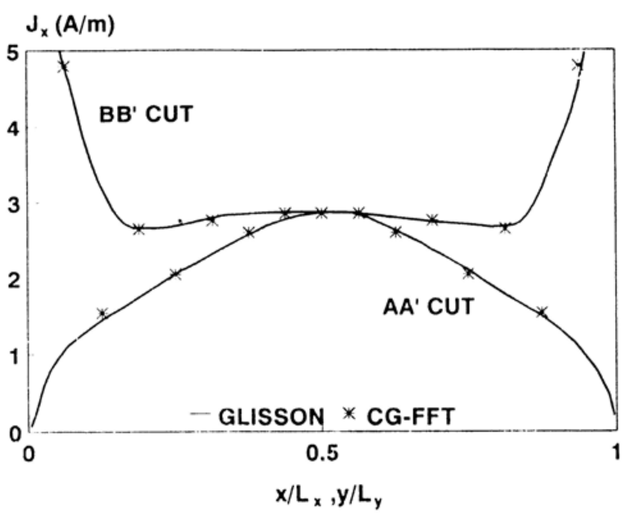


Figure 7 Comparison of the CG-FFT and MM solutions.

VIII. CONCLUSION

A general CG-FFT method of corrected current calculations based on the EFIE for numerical instabilities due to nearly resonant size has been given. An efficient numerical technique for implementing this correction, whenever the CG-FFT equation is being solved by a direct method, has also been demonstrated. Although the method given is iterative, it is expected to always stabilize the answer in three or less iterations. Furthermore, it has been shown that the CG-FFT method is an efficient numerical tool to analyze the electrodynamic behavior of a square conducting plate structure. However, the method presented here requires only about one-fourth as much computer time and appears to have better numerical stability properties. The method also appears to offer greater numerical stability in some types of problems.

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