Radiation and Scattering from Flat Conducting Plates of Resonant Size

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ملخص

استعملت الطريقة التكرارية لتوليفة تدرج المرافق وتحويله فورير السريعة لحساب تشتت المجال المغناطيسي والكهربائي الناتج عن مستوى موصل مربع الشكل موضوع في الفضاء الحر. إن كفاءة هذه الطريقة تكمن في العمليات الحسابية لتكاملات الطيات التي تمت باستخدام تحويلة فورير السريعة. وهذا مكن الطريقة من تقليل متطلبات زمن الوحدة المركزية للعمليات الحسابية (CPU) وخزن الذاكرة وذلك بترتيب القيم. كما أن هذه الطريقة قادرة على التعامل مع القطع المحيرة ذات الأشكال العشوائية، وذات فائدة لتحليل الأشكال التي لم يتم تحليلها سابقاً. لقد تم وباستعمال هذه الطريقة حساب مساحة مقطع الرادار للمستوى الموصل المربع والدائري ومقارنته مع القيم المقاسة.

ABSTRACT:

The purpose of this research is to introduce and applied the conjugate gradient fast Fourier transform (CG-FFT) method, and particularly to situate it within the electrical field integral equation (EFIE) of applied computational electromagnetics (ACE). The conjugate gradient fast Fourier transform (CG - FFT) - based iterative approach for computing the fields scattered for different conducting plates (square and circular disk) in free space is used. The efficiency of the CG - FFT method is due to the fact that the integrals of the convolutions are computed by means of FFTs. This makes it possible to reduce the central processing unit (CPU) time and memory storage requirements by order of magnitude. This method is also capable of handling patches that are lossy and have arbitrary shape, it is useful for analyzing configurations that may not have been analyzed previously. The radar cross section (RCS) of different conducting plates (square and circular disk) are calculated using this scheme and compared with results that available in the literature.

I. INTRODUCTION

The design of flat structures such as microstrip antennas, microwave integrated circuits (MIC) and microwave monolithic integrated circuits (MMIC), and feeding networks has become a very important topic in current electromagnetics. Due to the complexity of these devices, efficient numerical tools are required for their analysis. The method of moment (MOM), perhaps the most popular numerical tool for its flexibility to analze coplex geometries, becomes computationally too expensive to analze this class of problems (too much memory and central processing unit (CPU) time are required with ordinary computers). However, the CG-FFT method is a good and efficient alternative for analysis of this kinds of structures [1-4]. All the methods mentioned above have their strong and weak points, and it is practically impossible to establish which one of them is the best in general. All of them incorporate important features to reduce dramatically the computational cost involved in analyzing resonance-sized problems, especially if they are compared to traditional moment method (MM) schemes. The performance of any one of these methods will depend on the particular problem being considered. From a very general point of view, the CG-FFT method is ideal for periodic problems, because in these cases the discrete nature of the operator equation in the spectral domain greatly simplifies the computation of the Fourier transform by means of the FFT [4]. Problems having geometries with a body of revolution (BOR) symmetry will be included in the class of periodic problems because BORs are periodic for the revolution angle (the azimuthal coordinate if the BOR axis is located on the z-axis of a cylindrical coordinate system) [1]. The CG-FFT is also well suited to volumetric or surface problems that fill completely (or nearly) a rectangular parallelepiped in the case of three-dimensional (3D) problems, or a rectangular in the case of plate problems. For the CG-FFT to be efficient, the geometry of the problem must be well represented by means of a regular mesh.

Recent investigations to solve these problems in the resonance range have turned to iterative methods, one of the most promising being the conjugate gradient (CG) method [2-6]. The approach basically consists of solving the original integrodifferential equation by the conjugate gradient (CG) method. However, the fast Fourier transform (FFT) is utilized for efficient computations of certain terms required by the (CG) method. On the other hand, since the derivatives with respect to the spatial coordinates are replaced with simple multiplications in the transformed domain, some of the computational difficulties do not appear in this scheme. Finally, since the approach is iterative, it is possible to know the accuracy in the solution at each iteration [2].

This paper focuses on the RCS scattering problem from square and circular conducting plates. In Section II, CG method is briefly discussed. For completeness of this research, in Section III, a brief of the operator equations for the induced currents on square and circular disk are presented. The induced currents on the square conducting geometry are presented in [2]. In Section IV, numerical results for the induced currents on the circular disk are presented. The details of the application of the new approach to the problem of electromagnetic scattering is presented in Section V. Finally, in Section VI, numerical results for square conducting plate and circular disk are presented and compared with the available values that appear in the literature.

II. CG METHODS

The CG method is basically an iterative optimization procedure for which many applications have been found in areas such as optimal control, signal processing, numerical matrix algebra, universe problem analysis, computational geometry, and electromagnetics. The CG method is well suited to the treatment of operator equations such as the linear system of equations,

$$LI = Y \qquad (1)$$

where L is an integrodifferential operator, and I and Y are the unknown and excitation function vectors, respectively. The most general case is when L matrix is a nonsymmetric, complex value matrix (i.e., a non-Hermition matrix). In this case, L becomes a non-self - adjoint matrix. There are several algorithms that are used to apply the CG method in order to solve the operator equation (1), but in this paper, the CG algorithm for square conducting plate operators that appears in [2] can be considered. At each iteration of the CG algorithm it is required to compute LP or L^aP, where P is a known function and L^a is the adjoint operator of L. Usually, these operations are convolution integrals that can be performed efficiently using the fast Fourier transform (FFT) [1-4]. These convolutions involve the Green's functions as formulated in terms of the usual electric field equation.

Field equations aim to represent the space- and time-dependent responses of a linear field due to known excitations and subject to boundary conditions. The boundary conditions are generally imposed by the presence of arbitrary structures affecting the behavior of the fields, and the vector field solutions will be different depending on the geometry and physical properties of the structures. A general electromagnetic problem may be represented by a source located anywhere in space, which generates a known excitation. This excitation is applied to an environment and the modified fields will be intercepted by a receiver or scattered. The direct problem consists of finding the modified fields that will be intercepted by the receiver when both the sources and environment are known. The inverse problem, on the other hand, involves determining the geometry or physical characteristics (properties) of the environment when the sources and modified fields are both known.

This paper deals with the direct problem in which the environment consists of a flat (2D) conducting plate of arbitrary shape located anywhere in 3D free space. In this case, the physical properties are well known: a perfectly conducting body and free space. One of the most powerful approaches for solving electromagnetic problems is to find an integral equation that makes use of the simplest Green's function. It is well known that the solution to Maxwell's equations for a linear medium may be represented as the fields created by unit sources (dyadic Green's function). The original field can then be represented in terms of the overall excitation (an infinite sum of unit sources), thus giving rise to integral representations of the fields. Frequently, the general dyadic Green's function can be represented in terms of a scalar Green's function, which is much easier to obtain. Moreover, it is advantageous to reduce the original vector-field equations to scalar-wave equations and the corresponding solution to unit sources (scalar Green's function).

When the principles of the linearity and space invariance are applicable, we can write the fields as an integral of convolution between the source function and a Green's function, as the following expression shows for the case of the electric field created by the current on a conducting plate:

$$E(r) = L_{Dr} \int_{S'} G(r - r')J(r')ds' \qquad (2)$$

where the operator L_{Dr} can contain a derivative in the variable r, and S is the area of the plate. The Green's function G(r-r′) can also introduce some derivatives over the variable r′ of the source function, which is the current J(r′) in (2). It should be pointed out that the Green's function is written only in terms of the difference between the observation and source point vectors (r-r′). Now it is easy to identify these ACE cases with the linear and time-invariant (LTI) systems of signal processing. Employing the same terminology as in signal processing, we will use LTI to refer to linear and space-invariant problems like the one of (2).

III. THE DISCRETE OPERATOR EXPRESSIONS

The general problem to be solved is shown in Figure 1. A perfectly conducting plate of arbitrary geometry is illuminated in the direction of the local angles (ϕ^i,θ^i) by an incident field $\mathbf{E}^i$ created by a source current $\mathbf{J}^i$ anywhere in space. This field will induce currents on the plate **J**, which will generat a scattered field $\mathbf{E}^s$. At any point in space, the total field $\mathbf{E}^T$ will be the sum of the incident and scattered fields, assuming that no other sources exist. The scattered field will depend on the particular currents induced on the plate. Thus, the first step consists of obtaining these induced currents. In [4], the current equations for arbitrary flat conducting plate are presented. The metallization is enclosed in a rectangle of dimensions L_x and L_y, the maximum sizes of the flat conducting plase in the x- and y-coordinates. In this paper, we will be concerned with a formulation in the spatial domain because the expression of 3D Green's function for free space is will known and easy to evaluated [2].

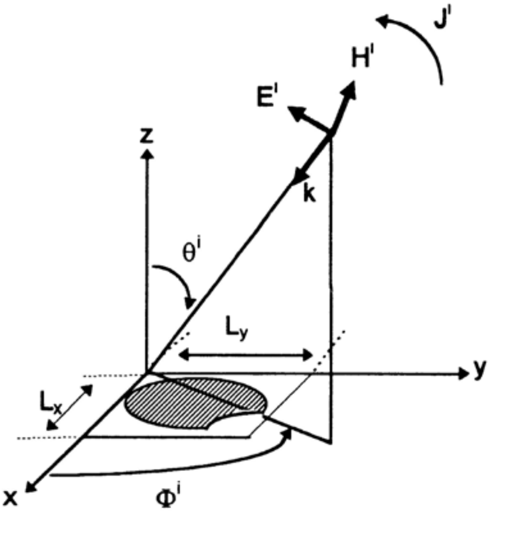

Figure 1. Schematic representation of the finite metallic plate problem.

Thus, the induced currents on the plate can be found by solving electric field integral equation (EFIE) (2) in terms of Green's function. In [2] we presented the full detailed formulation of the developed discrete operator equations. The currents on the flat conducting plate can be computed using the following discrete periodic operators:

$$V_{xx}^{D}[m,n] = k^{c}V_{x}^{pc}[m,n] + k^{q}V_{xx}^{pq}[m,n]$$

$$= \frac{\Delta x}{N_x^T N_y^T} FFT^{-1}\{\overset{\approx}{J}{}_x^P[p,q]\overset{\approx}{G}{}^P[p,q]\} \tag{3a}$$

$$+ \frac{1}{\Delta x N_x^T N_y^T} FFT^{-1}\{(F_x[p]-1)(1-F_x^*[p]\overset{\approx}{J}{}_x^P[p,q]\overset{\approx}{G}{}^P[p,q]\}$$

$$V_{xy}^{D}[m,n] = k^{q}V_{xy}^{pq}[m,n]$$

$$= \frac{1}{\Delta y N_x^T N_y^T} FFT^{-1}\{(F_x[p]-1)(1-F_y^*[q])\overset{\approx}{J}{}_y^P[p,q]\overset{\approx}{G}{}^P[p,q]\} \tag{3b}$$

$$V_{yx}^{D}[m,n] = k^{q}V_{yx}^{pq}[m,n]$$

$$= \frac{1}{\Delta x N_x^T N_y^T} FFT^{-1}\{(F_y[q]-1)(1-F_x^*[p])\overset{\approx}{J}{}_x^P[p,q]\overset{\approx}{G}{}^P[p,q]\} \tag{3c}$$

$$V_{yy}^{D}[m,n] = k^{c}V_{y}^{pc}[m,n] + k^{q}V_{yy}^{pq}[m,n]$$

$$= \frac{\Delta y}{N_x^T N_y^T} FFT^{-1}\{\overset{\approx}{J}{}_y^P[p,q]\overset{\approx}{G}{}^P[p,q]\} \tag{3d}$$

$$+ \frac{1}{\Delta y N_x^T N_y^T} FFT^{-1}\{(F_y[q]-1)(1-F_y^*[q]\overset{\approx}{J}{}_y^P[p,q]\overset{\approx}{G}{}^P[p,q]$$

where

$$N_x^T = 2(N_x + 1) \tag{4}$$

$$N_y^T = 2(N_y + 1) \tag{5}$$

In (3), the shift property of the discrete Fourier transform (DFT) has been employed to compute the finite differences of the capacitive terms [5]. The exponentials due to this property have been denoted by

$$F_x[p] = \exp\left(j\frac{2\pi}{N_x^T}p\right) \tag{6}$$

and

$$F_y[q] = \exp\left(j\frac{2\pi}{N_y^T}q\right) \tag{7}$$

$F_x^*[P]$ is the complex conjugate of $F_x[P]$, and $\tilde{\tilde{J}}_x^P$, $\tilde{\tilde{G}}^P$ indicate the periodic discrete Fourier transform of J_x^P and G^P, respectively.

The DFT of the Green's function directly obtained by using the spectral domain discretization procedure- defined in [2]. In the first place, no zero padding will be required for problems with individual structures also when forming the periodic function. This represents an advantage in both CPU time and memory storage requirements because the discrete transform will be a DFTN, where N is the number of samples used to represent the course function, and since there is no zero padding, it is obvious that it will not be necessary to double the sizes of the matrices.

The pair of functions, rooftop and razor blade, have been used throughout this research for the analysis of electrodynamic problems for one major reason and several minor reasons are that they are easy to draw and to treat mathematically, they provide the paper with uniform presentation, and as mentioned in [2], they can easily be changed if necessary.

The most important reason, as will be seen, is related to the solution of electrodynamic problems by using electric field integral equation in its mixed potential formulation, where two source functions must be handled: the current and its divergence (the charge). Theoretically, this would involve computing two potential functions: the magnetic vector potential A and the scalar electric potential ϕ. The major advantage of using rooftops and razor blades is that they allow these problems to be solved by computing the convolution of only one potential function. This reduces memory storage requirements and CPU time by nearly half.

The application of the CG Section II requires the use of the adjoint operator defined in terms of the inner product [8,9]. To apply the conjugate gradient method, we need to construct an adjoint aperator and defined an inner product. Consider FFT as a matrix operators; the conplex conjugate transpose of the FFT operation is simply equal to the inverse FFT operation and vice versa. The full detailed formulation of the adjoint operator equations are developed in [2]. For completeness of this research, here we summarize the final periodic equations for the adjoint operators, in view of this, the adjoint operators for (3) can be written as:

$$V_{xx}^{DA}[m,n] = \frac{k^{c^*}\Delta x}{N_x^T N_y^T} FFT^{-1}\{\overset{\approx P}{J}_x[p,q]\overset{\approx P^*}{G}[p,q]\}$$
$$+ \frac{k^{q^*}}{\Delta y N_x^T N_y^T} FFT^{-1}\{(F_x^*[p]-1)(1-F_x[p])\overset{\approx P}{J}_x[p,q]\overset{\approx P^*}{G}[p,q]\} \tag{8a}$$

$$V_{xy}^{DA}[m,n] = \frac{k^{q^*}}{\Delta x N_x^T N_y^T} \bullet$$
$$FFT^{-1}\{(F_y^*[q]-1)(1-F_x[p])\overset{\approx P}{J}_y[p,q]\overset{\approx P^*}{G}[p,q]\} \tag{8b}$$

$$V_{yx}^{DA}[m,n] = \frac{k^{q^*}}{\Delta y N_x^T N_y^T} \bullet$$
$$FFT^{-1}\{(F_x^*[p]-1)(1-F_y[q])\overset{\approx P}{J}_x[p,q]\overset{\approx P^*}{G}[p,q]\} \tag{8c}$$

$$V_{yy}^{DA}[m,n] = \frac{k^{c^*}\Delta y}{N_x^T N_y^T} FFT^{-1}\{\overset{\approx P}{J}_y[p,q]\overset{\approx P^*}{G}[p,q]\}$$
$$+ \frac{k^{q^*}}{\Delta y N_x^T N_y^T} FFT^{-1}\{(F_y^*[q]-1)(1-F_y[q])\overset{\approx P}{J}_y[p,q]\overset{\approx P^*}{G}[p,q]\} \tag{8d}$$

The values of the constants are

$$k^c = j\omega\mu_0 \tag{9}$$

$$k^{q} = -\frac{1}{j\omega\varepsilon_0} \qquad (10)$$

IV. RESULTS FOR INDUCED CURRENTS

By using (3) and (8) for the direct and adjoint operators, respectively, in CG algorithm developed in [2], we are able to obtain the induced currents on the flat conducting plates structures. This Section presents the results obtained for the circular disk, while in [2] we presented the induced currents for the square conducting plate.

Another important structure is the circular metallic disk. The problem is illustrated in Figure (2), where an x-polarized plane wave is normal incident upon the structure.

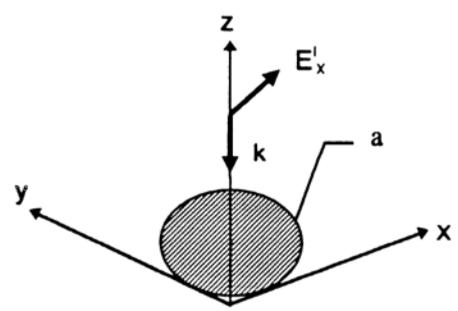

Figure 2 Schematic representation of the circular disk problem.

The x-and y-components of the current are presented in Figures (3a,b), respectively. The radius of the disk is denoted by a. This is a good test of the convergence of the method when the structure becomes large in terms of $\lambda(k_0a = 10)$.

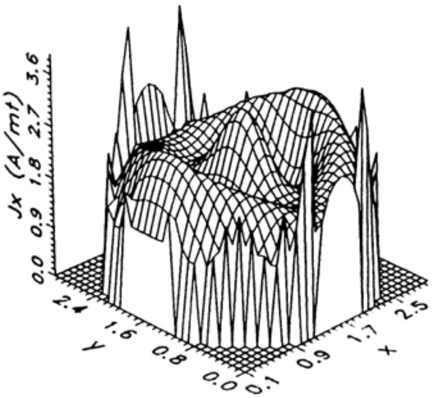

Figure 3(a) 2D representation of the x-component of the current.

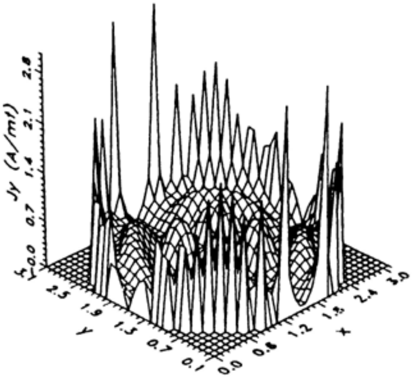

Figure 3(b) 2D representation of the y-component of the current.

The number of samples used to represent the currents was $N_x = N_y = 31$, and the time required to get a relative error of 5.E-3 was a bout 3 minutes on a PC-486. In Section IV, we will compare the RCS obtained by using these currents with other results available in the literature.

V. APPLICATIONS TO RADIATION SCATTERING PROBLEMS

The equivalent currents on the conducting plate are found by using (3) and (8) in the CG algorithm developed in [2]. Once the equivalent currents on the plate have been found, the radiated fields can be obtained from

$$\mathbf{E} = -j\omega\mathbf{r} \times (\mathbf{A} \times \mathbf{r}) \tag{11}$$

where

$$A = \frac{\mu_0}{4\pi}\frac{\exp[-jk_0 r]}{r}\int_s J(r')\exp[jk_0 rr']ds \tag{12}$$

$$\mathbf{r} = x\mathbf{x} + y\mathbf{y} + z\mathbf{z} \tag{13a}$$

$$\mathbf{r}' = x'\mathbf{x} + y'\mathbf{y} \tag{13b}$$

and the unit position vector can be described by θ and ϕ as

$$\mathbf{r} = \sin\theta\cos\phi\mathbf{x} + \sin\theta\sin\phi\mathbf{y} + \cos\theta\mathbf{z} \tag{14}$$

The geometrical parameters of the problem are depicted in Figure 4, with θ and ϕ denoting the angles that define the position vector r in a spherical coordinate system.

Substituting (13b) and (14) into (12) , we have

$$\mathbf{A} = \frac{\mu_0}{4\pi}\frac{\exp[-jk_0 r]}{r}\int_0^{L_x}\int_0^{L_y} J(\mathbf{r}')\exp[jk_0 x'\sin\theta\cos\phi] \cdot \exp[jk_0 y'\sin\theta\sin\phi]\,dx'\,dy' \tag{15}$$

The integral in (15) is the continuous Fourier transform CFT of the currents considering the spectral variables

$$k_x = k_0 \sin\theta\cos\phi \tag{16a}$$

$$k_y = k_0 \sin\theta\sin\phi \tag{16b}$$

The local plane wave behavior of the field at the observation point **r** is described by E_θ and E_ϕ components.

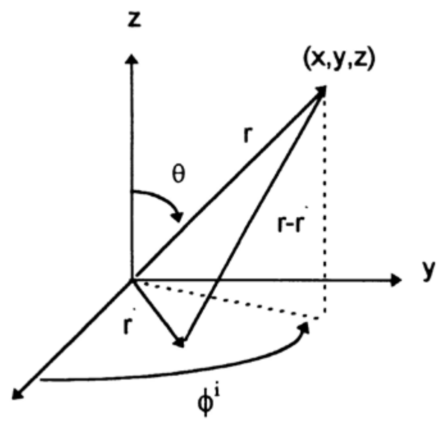

Figure 4 Parameters considered in the radiation problem.

By using the corresponding transformations, E_θ and E_ϕ can be written in terms of the CFTs of J_x and J_y:

$$\mathbf{E} = E_\theta \hat{\theta} + E_\phi \hat{\phi} \tag{17a}$$

$$E_\theta = E_0 \,[\, \tilde{\tilde{J}}_x (k_x,k_y) \cos\theta \cos\phi + \tilde{\tilde{J}}_y (k_x,k_y) \cos\theta \sin\phi] \tag{17b}$$

$$E_\phi = E_0 [\, \tilde{\tilde{J}}_y (k_x,k_y) \cos\phi - \tilde{\tilde{J}}_x (k_x,k_y) \sin\theta] \tag{17c}$$

$$E_0 = -\frac{j\omega\mu_0}{4\pi}\frac{\exp(-jk_0 r)}{r} \tag{17d}$$

The RCS is defined in terms of the scattered fields by

$$\sigma = 10\log\left(4\pi r^2 \frac{|E_\theta|^2 + |E_\phi|^2}{|E^i|^2}\right) \tag{18}$$

where E_θ and E_ϕ have already been computed in (17b,17c) and E^i is the amplitude of the incident plane wave.

VI. NUMERICAL RESULTS FOR SCATTERING FROM DIFFERENT FLAT PLATE GEOMETRIES

A. Square Plate

This structure is shown in Figure 5. As an example we consider a 3.0 λ square plate irradiated by a normally x-polarized plane wave incident field intensity. The results shown in Figure 6 correspond to the RCS as a function of the angle θ. The number of samples used in $N_x = N_y = 63$ and the numerical results are compared with measurements obtained from [9]. Result of RCS has been obtained using $2 \times 63 \times 64 = 8064$ rooftops. The CPU time with a VAX 750 $\times$ FPS 164 was 6 min.

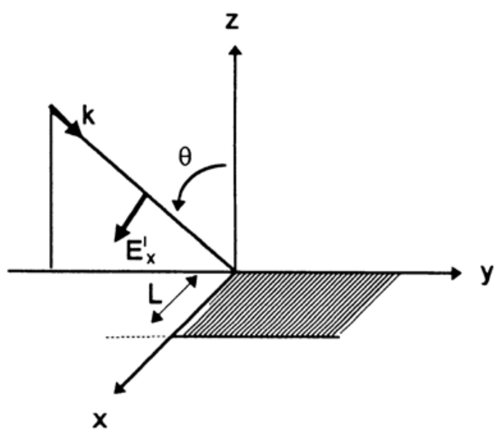

Figure 5 Incident wave of the square problem.

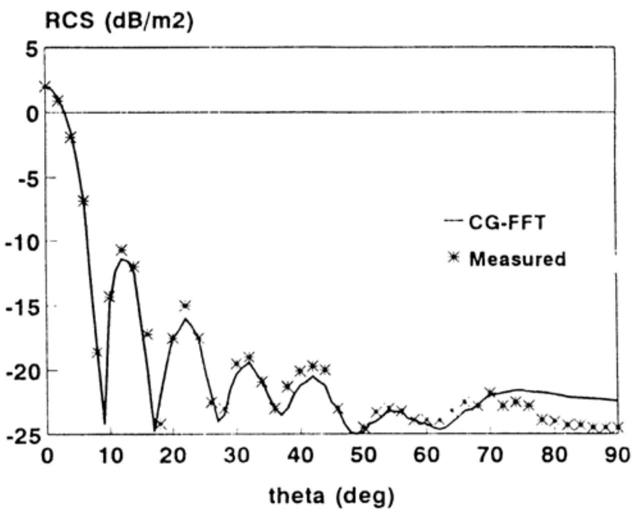

Figure 6 Radar cross section versus aspect angle of a square plate
(L=3λ, f_0 = 10 GHz).

B. Circular Disk

An x-polarized plane wave with normal incidence on a circular disk is depicted in Figure 7. The radius of the disk is denoted by a. RCS results ($\theta^i = \theta = 0$ deg) as a function of disk size are shown in Figure 8. The number of samples used to represent the currents varies depending on the size of the disk. The numerical results are compared with those presented by Hodge in [11].

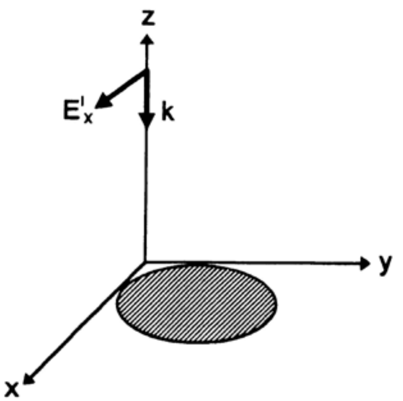

Figure 7 Incident wave of the circular disk problem.

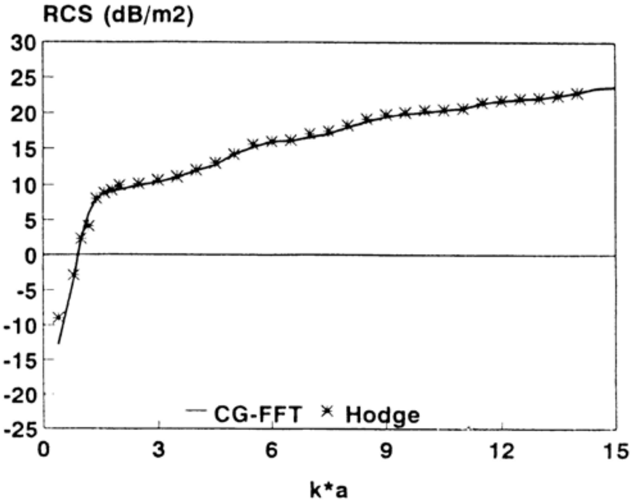

Figure 8 Radar cross section versus size of a disk of raduis a.

VII. CONCLUSION

In this paper we have shown that the electric field integral equation may be accurately solved by the method of CG-FFT using rooftops function as basis function and using the razor blade function for testing function to smooth the fields in which the second derivative occurs. Furthermore, the method of conjugate gradient along with the fast Fourier technique is utilized to solve for the current distribution on conducting plate electrically square and circular disk without running into numerical instability. These currents then be used to obtain any other parameters of interest such as radiated (scattered) fields and RCS. Therefore, the CG-FFT method is an efficient numerical tool to analyze the electrodynamics behavior of the square and circular disk conducting plate.

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